AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 1

Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Answer **all** questions in the spaces provided.

1 Find

$$\sum_{r=1}^{20} (r^2 - 2r)$$

Circle your answer.

[1 mark]

2450 2660 5320 43680

2 Given that $z = 1 - 3\mathrm{i}$ is one root of the equation $z^2 + pz + r = 0$, where p and r are real, find the value of r.

Circle your answer.

[1 mark]

-8 -2 6 10

Do not write outside the box

3 The curve C has polar equation

$$r^2 \sin 2\theta = 4$$

Find a Cartesian equation for C.

Circle your answer.

[1 mark]

$y = 2x$ $\quad$ $y = \frac{x}{2}$ $\quad$ $y = \frac{2}{x}$ $\quad$ $y = 4x$

Turn over for the next question

Do not write outside the box

4 Show that the solutions to the equation

$$3\tanh^2 x - 2\operatorname{sech} x = 2$$

can be expressed in the form

$$x = \pm \ln (a + \sqrt{b})$$

where a and b are integers to be found.

You may use without proof the result $\cosh^{-1} y = \ln (y + \sqrt{y^2 - 1})$

[5 marks]

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Turn over for the next question

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5 The matrix $\mathbf{M}$ is defined by $\mathbf{M} = \begin{bmatrix} 3 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Prove by induction that $\mathbf{M}^n = \begin{bmatrix} 3^n & 3^n - 1 & -3^n + 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for all integers $n \geq 1$

[5 marks]

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Turn over for the next question

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6 (a) Show that the equation

$$(2z - z^*)^* = z^2$$

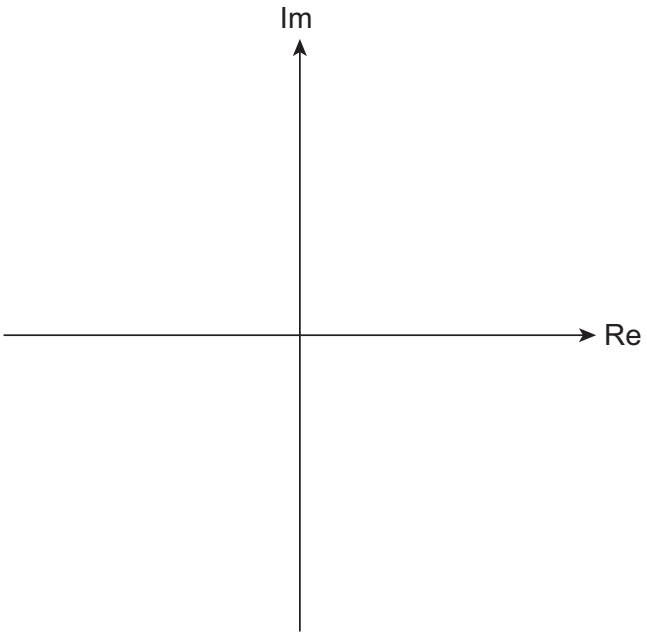
has exactly **four** solutions and state these solutions.

[7 marks]

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6 (b) (i) Plot the four solutions to the equation in part **(a)** on the Argand diagram below and join them together to form a quadrilateral with one line of symmetry.

[2 marks]



6 (b) (ii) Show that the area of this quadrilateral is $\frac{\sqrt{15}}{2}$ square units.

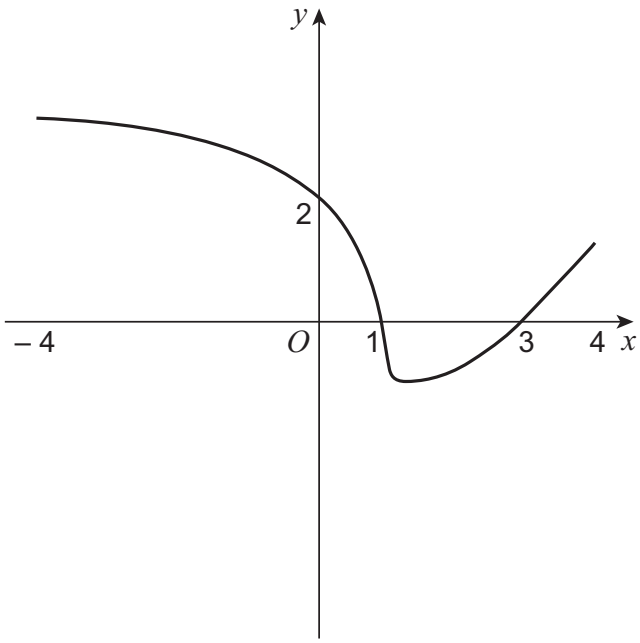
[1 mark]

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7 The diagram below shows the graph of $y = \mathrm{f}(x) \quad (-4 \leq x \leq 4)$

The graph meets the x-axis at $x = 1$ and $x = 3$

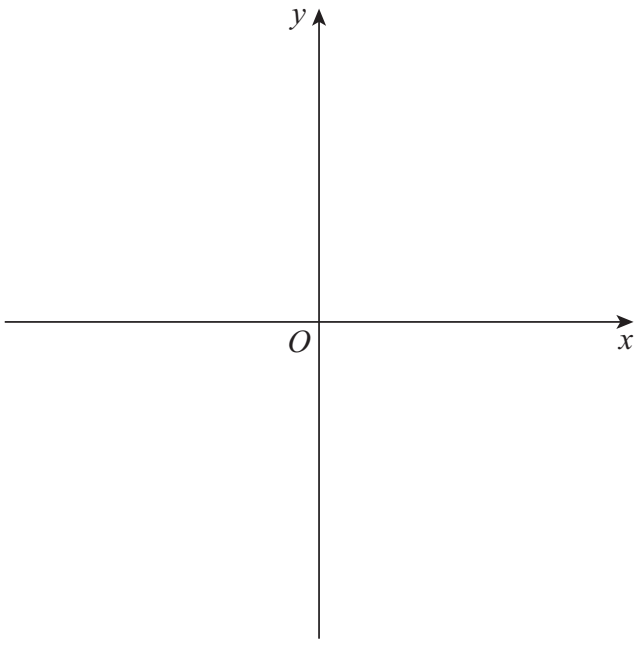
The graph meets the y-axis at $y = 2$



7 (a) Sketch the graph of $y = |\mathrm{f}(x)|$ on the axes below.

Show any axis intercepts.

[2 marks]

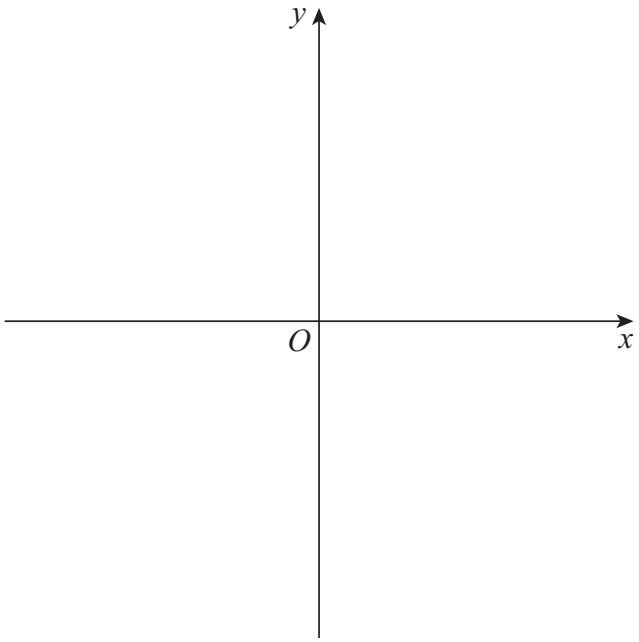


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7 (b) Sketch the graph of $y = \dfrac{1}{\mathrm{f}(x)}$ on the axes below.

Show any axis intercepts and asymptotes.

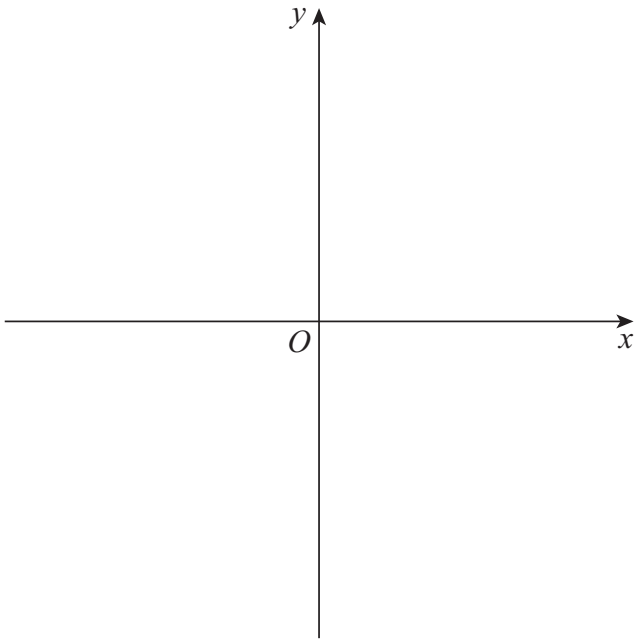
[3 marks]



7 (c) Sketch the graph of $y = \mathrm{f}(|x|)$ on the axes below.

Show any axis intercepts.

[2 marks]



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8 A particle of mass 4 kg moves horizontally in a straight line.

At time t seconds the velocity of the particle is v m s^{-1}

The following horizontal forces act on the particle:

- a constant driving force of magnitude 1.8 newtons
- another driving force of magnitude $30\sqrt{t}$ newtons
- a resistive force of magnitude $0.08v^2$ newtons

When $t = 70$, $v = 54$

Use Euler's method with a step length of 0.5 to estimate the velocity of the particle after 71 seconds.

Give your answer to **four** significant figures.

[6 marks]

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Turn over for the next question

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9 Use l'Hôpital's rule to show that

$$\lim_{x \to \infty} (xe^{-x}) = 0$$

Fully justify your answer.

[4 marks]

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10 Evaluate the improper integral

$$\int_0^8 \ln x \, \mathrm{d}x$$

showing the limiting process.

[6 marks]

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11 The line L_1 has equation $\mathbf{r} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

The line L_2 has equation $\mathbf{r} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

11 (a) Find the acute angle between the lines L_1 and L_2, giving your answer to the nearest 0.1°

[3 marks]

11 (b) The lines L_1 and L_2 lie in the plane Π_1

11 (b) (i) Find the equation of Π_1, giving your answer in the form $\mathbf{r}.\mathbf{n} = d$

[4 marks]

Do not write outside the box

11 (b) (ii) Hence find the shortest distance of the plane Π_1 from the origin.

[1 mark]

11 (c) The points $A(4, -1, -1)$, $B(1, 5, -7)$ and $C(3, 4, -8)$ lie in the plane Π_2

Find the angle between the planes Π_1 and Π_2, giving your answer to the nearest 0.1°

[4 marks]

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12 The matrix $\mathbf{A} = \begin{bmatrix} 1 & 5 & 3 \\ 4 & -2 & p \\ 8 & 5 & -11 \end{bmatrix}$, where p is a constant.

12 (a) Given that **A** is a non-singular matrix, find $\mathbf{A}^{-1}$ in terms of p.

State any restrictions on the value of p.

[6 marks]

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12 (b) The equations below represent three planes.

$$\begin{aligned} x + 5y + 3z &= 5 \\ 4x - 2y + pz &= 24 \\ 8x + 5y - 11z &= -30 \end{aligned}$$

12 (b) (i) Find, in terms of p, the coordinates of the point of intersection of the three planes.

[4 marks]

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12 (b) (ii) In the case where $p = 2$, show that the planes are mutually perpendicular.

[4 marks]

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13 The transformation S is represented by the matrix $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

The transformation T is a translation by the vector $\begin{bmatrix} 0 \\ -5 \end{bmatrix}$

Kamla transforms the graphs of various functions by applying first S, then T.

Leo says that, for some graphs, Kamla would get a different result if she applied first T, then S.

Kamla disagrees.

State who is correct.

Fully justify your answer.

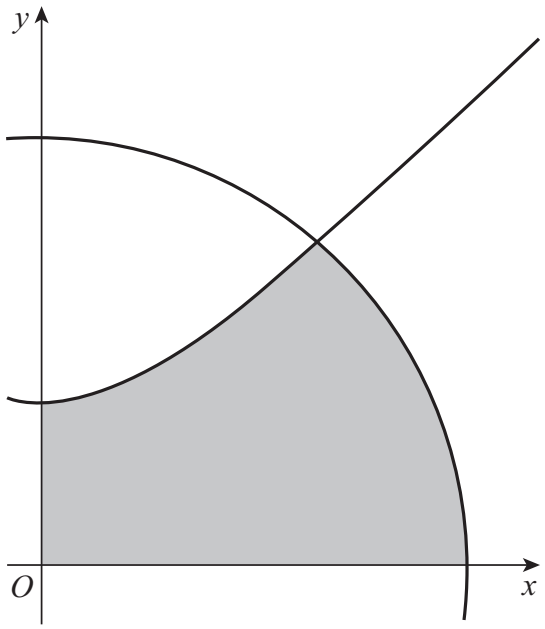
[3 marks]

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14 The hyperbola H has equation $y^2 - x^2 = 16$

The circle C has equation $x^2 + y^2 = 32$

The diagram below shows part of the graph of H and part of the graph of C.



Show that the shaded region in the first quadrant enclosed by H, C, the x-axis and the y-axis has area

$$\frac{16\pi}{3} + 8\ln\left(\frac{\sqrt{2}+\sqrt{6}}{2}\right)$$

[12 marks]

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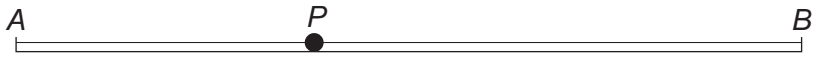
15 **In this question use $g = 9.8\,\text{m}\,\text{s}^{-2}$**

A particle P of mass m is attached to two light elastic strings, AP and BP.

The other ends of the strings, A and B, are attached to fixed points which are 4 metres apart on a rough horizontal surface at the bottom of a container.

The coefficient of friction between P and the surface is 0.68

- When the extension of string AP is e_A metres, the tension in AP is $24me_A$
- When the extension of string BP is e_B metres, the tension in BP is $10me_B$
- The natural length of string AP is 1 metre
- The natural length of string BP is 1.3 metres



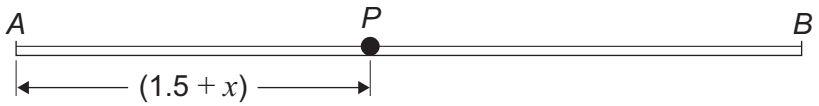
15 (a) Show that when $AP = 1.5$ metres, the tension in AP is equal to the tension in BP.

[1 mark]

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15 (b) P is held at the point between A and B where $AP = 1.9$ metres, and then released from rest.

At time t seconds after P is released, $AP = (1.5 + x)$ metres.



Show that when P is moving towards A,

$$\frac{d^2x}{dt^2} + 34x = 6.664$$

[3 marks]

Question 15 continues on the next page

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15 (c) The container is then filled with oil, and P is again released from rest at the point between A and B where $AP = 1.9$ metres.

At time t seconds after P is released, the oil causes a resistive force of magnitude $10mv$ newtons to act on the particle, where $v\,\text{m}\,\text{s}^{-1}$ is the speed of the particle.

Find x in terms of t when P is moving towards A.

[9 marks]

END OF QUESTIONS

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