AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 3 Discrete

Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics **or** Statistics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 Which of the following statements about critical path analysis is always true?

Tick (✓) **one** box.

[1 mark]

All activity networks have exactly one critical path. ☐

All critical activities have a non-zero float. ☐

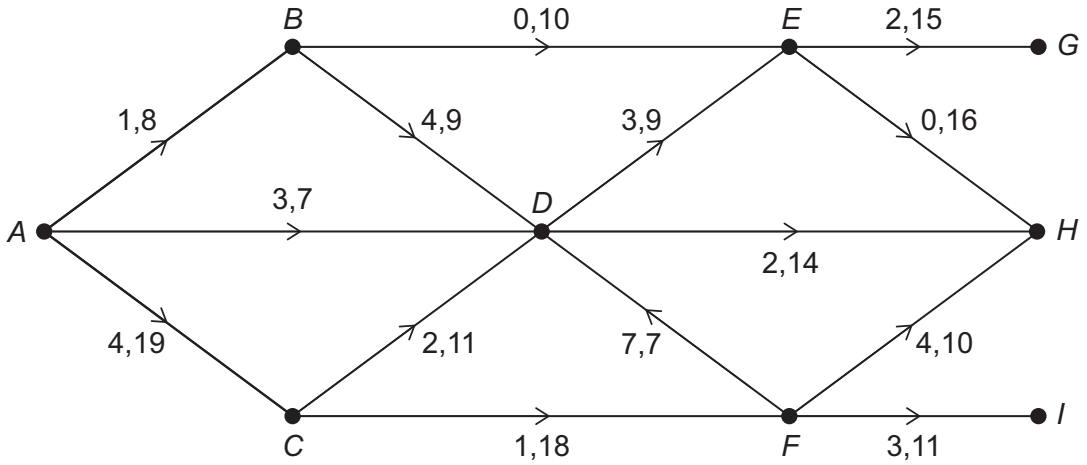
The first activity in a critical path has an earliest start time of zero. ☐

A delay on a critical activity may not delay the project. ☐

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2 The network below represents a system of pipes.

The numbers on each arc represent the lower and upper capacity for each pipe.



Find the value of the cut $\{A, B, C, D, E\}$ $\{F, G, H, I\}$.

Circle your answer.

[1 mark]

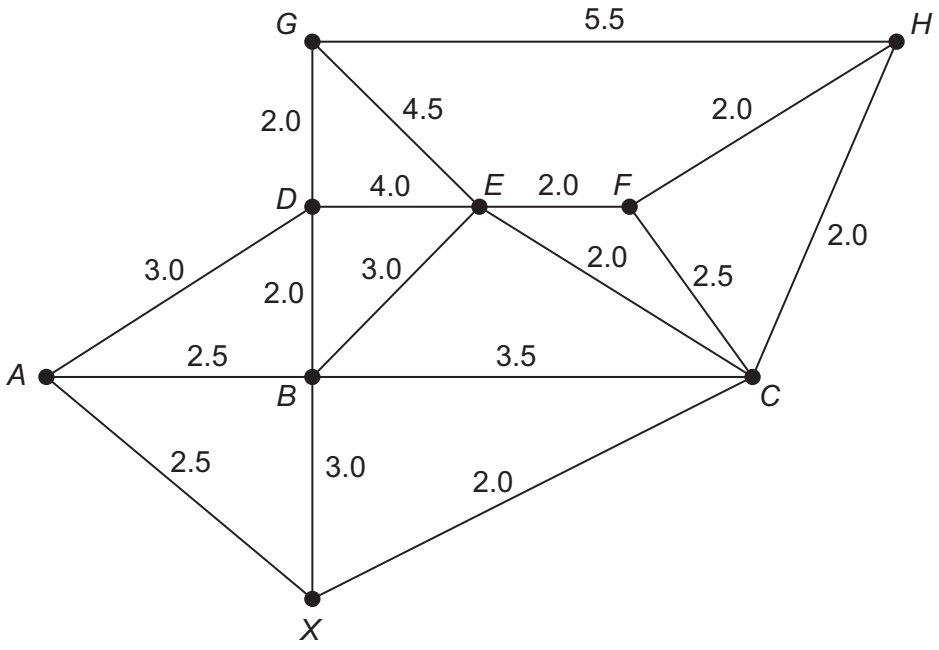
56 58 63 70

Turn over for the next question

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3 A mining company wants to open a new mine in an area where the ground contains a precious metal.

The mining company has carried out a survey of the area. The network below shows nodes which represent the entrance to the new mine, *X*, and the 8 ventilation shafts, *A*, *B*, …, *H*, which have been installed to prevent the build up of dangerous gases underground.



Each arc represents a possible underground tunnel which could be mined.

The weight on each arc represents the estimated amount of precious metal in that possible underground tunnel in tonnes.

Due to geological reasons, the mining company can only create 8 underground tunnels.

All 8 ventilation shafts must be accessible from the entrance of the mine.

3 (a) (i) The mining company wants to **maximise** the amount of precious metal it can extract from the new mine.

Determine the tunnels the mining company should use.

[2 marks]

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3 (a) (ii) Estimate the maximum amount of precious metal the mining company can extract from the new mine.

[2 marks]

3 (b) Comment on why the maximum amount of precious metal the mining company can extract from the new mine may be different from your answer to part **(a)(ii)**.

[2 marks]

3 (c) Before the mining company begins work on the new mine, a government survey prevents the mining company drilling the tunnel represented by *CF*.

Determine the effect, if any, the government survey has on your answers to part **(a)(i)** and part **(a)(ii)**.

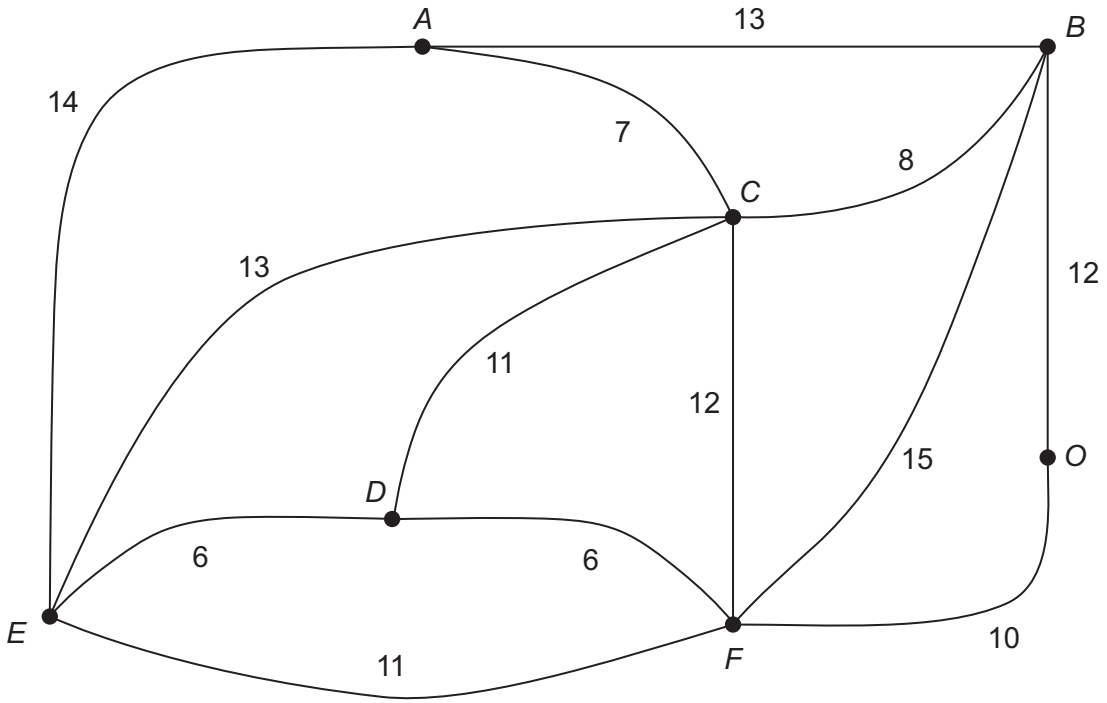
[2 marks]

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4 Derrick, a tanker driver, is required to deliver fuel to 6 different service stations *A*, *B*, *C*, *D*, *E* and *F*.

Derrick needs to begin and finish his delivery journey at the refinery *O*.

The distances, in miles, between the 7 locations which have a direct road between them are shown in the network below.



Derrick spends 30 minutes at each service station to complete the fuel delivery.

When driving, the tanker travels at an average speed of 40 miles per hour.

The minimum total time that it takes Derrick to travel to and deliver fuel to all 6 service stations, starting and finishing at the refinery, is T minutes.

4 (a) Using the nearest neighbour algorithm starting from the refinery, find an upper bound for T

[5 marks]

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4 (b) Before setting off to make his fuel deliveries, Derrick is notified that, due to a low bridge, the road represented by *CE* is not suitable for tankers to travel along.

State, with a reason, the effect this new information has on your answer to part **(a)**.

[2 marks]

Turn over for the next question

Do not write outside the box

5 (a) Describe the conditions necessary for a set of elements, S, under a binary operation * to form a group.

[3 marks]

5 (b) In the multiplicative group of integers modulo 13, the group G is defined as

$$G = (\langle 10 \rangle, \times_{13})$$

5 (b) (i) Explain why G is an abelian group.

[1 mark]

5 (b) (ii) Find the order of G.

[2 marks]

Do not write outside the box

5 (c) State the identity element of G and prove it is an identity element.

Fully justify your answer.

[3 marks]

5 (d) Find all the proper non-trivial subgroups of G, giving your answers in the form $(\langle g \rangle, \times_{13})$, where g is an integer less than 13

[2 marks]

Turn over for the next question

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6 (a) A connected planar graph has $(x+1)^2$ vertices, $(25+2x-2y)$ edges and $(y-1)^2$ faces, where $x>0$ and $y>0$

Find the possible values for the number of vertices, edges and faces for the graph.

[6 marks]

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6 (b) Explain why K_6, the complete graph with 6 vertices, is not planar.

Fully justify your answer.

[2 marks]

Turn over for the next question

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7 Avon and Roj play a zero-sum game.

The game is represented by the following pay-off matrix for Avon.

		Roj		
	Strategy	$\mathbf{R_1}$	$\mathbf{R_2}$	$\mathbf{R_3}$
Avon	$\mathbf{A_1}$	3	1	4
	$\mathbf{A_2}$	2	3	2
	$\mathbf{A_3}$	6	2	1

The zero-sum game does not have a stable solution.

7 (a) Avon wants to know the optimal mixed strategy he should play in order to maximise his winnings from the game.

Avon defines the following variables:

$p_1 =$ the probability of Avon playing strategy $\mathbf{A_1}$

$p_2 =$ the probability of Avon playing strategy $\mathbf{A_2}$

$p_3 =$ the probability of Avon playing strategy $\mathbf{A_3}$

$v =$ the value of the game for Avon

Formulate Avon's situation as a linear programming problem.

[4 marks]

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7 (b) (i) Using your answer to part **(a)**, complete the initial Simplex tableau for Avon's situation in the grid below.

[3 marks]

P	v	p_1	p_2	p_3					**value**

7 (b) (ii) Hence, perform **one** iteration of the Simplex algorithm, showing your answer on the grid below.

[3 marks]

P	v	p_1	p_2	p_3					**value**

Question 7 continues on the next page

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7 (c) Avon performs further iterations of the Simplex algorithm until an optimal solution is reached.

The grid below shows part of Avon's final Simplex tableau.

p_1	p_2	p_3	**value**
1	0	0	0.25
0	1	0	0.70
0	0	1	0.05

7 (c) (i) Find the optimal mixed strategy for Avon.

[1 mark]

7 (c) (ii) Find the value of the game for Avon.

[1 mark]

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7 (d) Roj thinks that his best outcome from the game is to play strategy $\mathbf{R_2}$ each time.

Avon notices that Roj always plays strategy $\mathbf{R_2}$ and Avon wants to use this knowledge to maximise his expected pay-off from the game.

Explain how your answer to part **(c)(i)** should change and find Avon's maximum expected pay-off from the game.

[2 marks]

END OF QUESTIONS

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