AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 2

Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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Answer **all** questions in the spaces provided.

1 Find the imaginary part of

$$\frac{5+\mathrm{i}}{1-\mathrm{i}}$$

Circle your answer.

[1 mark]

-3 -2 2 3

2 Find the mean value of the function $\mathrm{f}(x) = 10x^4$ between $x = 0$ and $x = a$

Circle your answer.

[1 mark]

$10a^3$ $40a^3$ $2a^4$ $4a^5$

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3 The roots of the equation $x^2 - px - 6 = 0$ are α and β

Find $\alpha^2 + \beta^2$ in terms of p

Circle your answer.

[1 mark]

$p^2 - 6$ $p^2 + 6$ $p^2 - 12$ $p^2 + 12$

4 Which of the following graphs intersects the graph of $y = \sinh x$ at exactly one point?

Circle your answer.

[1 mark]

$y = \text{cosech}\, x$ $y = \cosh x$ $y = \coth x$ $y = \text{sech}\, x$

Turn over for the next question

Do not write outside the box

5 Prove by induction that, for all integers $n \geq 1$,

$$\sum_{r=1}^{n} r^3 = \left\{\frac{1}{2}n(n+1)\right\}^2$$

[4 marks]

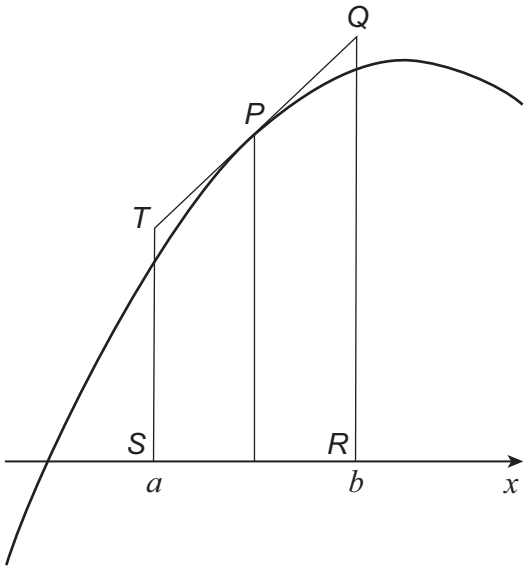
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6 The diagram below shows part of the graph of $y = \mathrm{f}(x)$

The line *TPQ* is a tangent to the graph of $y = \mathrm{f}(x)$ at the point $P\left(\frac{a+b}{2}, \mathrm{f}\left(\frac{a+b}{2}\right)\right)$

The points $S(a, 0)$ and *T* lie on the line $x = a$

The points *Q* and $R(b, 0)$ lie on the line $x = b$



Sharon uses the mid-ordinate rule with one strip to estimate the value of the integral $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$

By considering the area of the trapezium *QRST*, state, giving reasons, whether you would expect Sharon's estimate to be an under-estimate or an over-estimate.

[3 marks]

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7 The function f is defined by

$$\mathrm{f}(x) = \frac{ax - 5}{2x + b} \qquad x \in \mathbb{R},\ x \neq \frac{9}{2}$$

where a and b are integers.

The graph of $y = \mathrm{f}(x)$ has asymptotes $x = \frac{9}{2}$ and $y = 3$

7 (a) Find the value of a and the value of b

[2 marks]

7 (b) Solve the inequality

$$\mathrm{f}(x) \leq x + 2$$

Fully justify your answer.

[6 marks]

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Turn over for the next question

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8 (a) The function f is defined as $\mathrm{f}(x) = \sec x$

8 (a) (i) Show that $\mathrm{f}^{(4)}(0) = 5$

[4 marks]

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8 (a) (ii) Hence find the first three non-zero terms of the Maclaurin series for $\mathrm{f}(x) = \sec x$

[2 marks]

8 (b) Prove that

$$\lim_{x \to 0} \left(\frac{\sec x - \cosh x}{x^4} \right) = \frac{1}{6}$$

[4 marks]

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9 (a) A curve passes through the point (5, 12.3) and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (x^2 - 9)^{\frac{1}{2}} + \frac{2xy}{x^2 - 9} \qquad x > 3$$

Use Euler's step by step method once, and then the midpoint formula

$$y_{r+1} = y_{r-1} + 2h\mathrm{f}(x_r, y_r), \quad x_{r+1} = x_r + h$$

once, each with a step length of 0.1, to estimate the value of y when $x = 5.2$

Give your answer to six significant figures.

[4 marks]

9 (b) (i) Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (x^2 - 9)^{\frac{1}{2}} + \frac{2xy}{x^2 - 9} \qquad (x > 3)$$

[6 marks]

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9 (b) (ii) Given that y satisfies the differential equation in part **(b)(i)** and that $y = 12.3$ when $x = 5$, find the value of y when $x = 5.2$

Give your answer to six significant figures.

[3 marks]

9 (c) Comment on the accuracy of your answer to part **(a)**.

[1 mark]

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10 The curve C_1 has equation

$$\frac{x^2}{25} - \frac{y^2}{4} = 1$$

The curve C_2 has equation

$$x^2 - 25y^2 - 6x - 200y - 416 = 0$$

10 (a) Find a sequence of transformations that maps the graph of C_1 onto the graph of C_2

[4 marks]

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10 (b) Find the equations of the asymptotes to C_2

Give your answers in the form $ax + by + c = 0$ where a, b and c are integers.

[3 marks]

Turn over for the next question

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11 (a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$\mathbf{M} = \begin{bmatrix} \frac{5}{2} & -\frac{3}{2} \\ -\frac{3}{2} & \frac{13}{2} \end{bmatrix}$$

[5 marks]

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11 (b) (i) Describe how the directions of the invariant lines of the transformation represented by **M** are related to each other.

Fully justify your answer.

[2 marks]

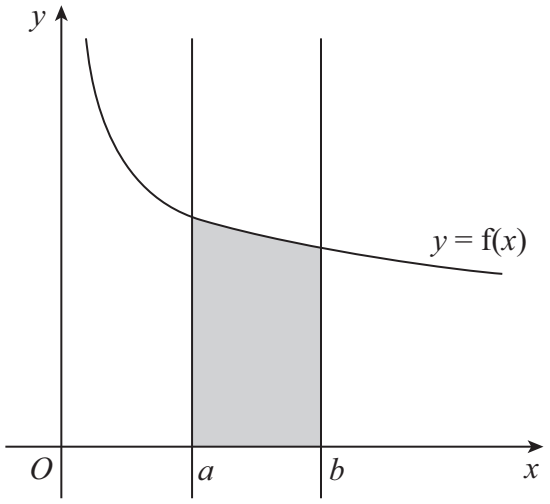
11 (b) (ii) Describe fully the transformation represented by **M**

[2 marks]

Turn over for the next question

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12 The shaded region shown in the diagram below is bounded by the x-axis, the curve $y = \mathrm{f}(x)$, and the lines $x = a$ and $x = b$



The shaded region is rotated through 2π radians about the x-axis to form a solid.

12 (a) Show that the volume of this solid is

$$\pi \int_a^b (\mathrm{f}(x))^2 \, \mathrm{d}x$$

[4 marks]

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12 (b) In the case where $a = 1$, $b = 2$ and

$$f(x) = \frac{x + 3}{(x + 1)\sqrt{x}}$$

show that the volume of the solid is

$$\pi\left(\ln\left(\frac{2^m}{3^n}\right) - \frac{2}{3}\right)$$

where m and n are integers.

[7 marks]

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13 (a) The matrix **A** represents a reflection in the line $y = mx$, where m is a constant.

Show that $\mathbf{A} = \left(\frac{1}{m^2 + 1}\right)\begin{bmatrix} 1 - m^2 & 2m \\ 2m & m^2 - 1 \end{bmatrix}$

You may use the result in the formulae booklet.

[5 marks]

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13 (b) The matrix **B** is defined as $\mathbf{B} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Show that $(\mathbf{BA})^2 = k\mathbf{I}$

where $\mathbf{I}$ is the 2×2 identity matrix and k is an integer.

[3 marks]

Question 13 continues on the next page

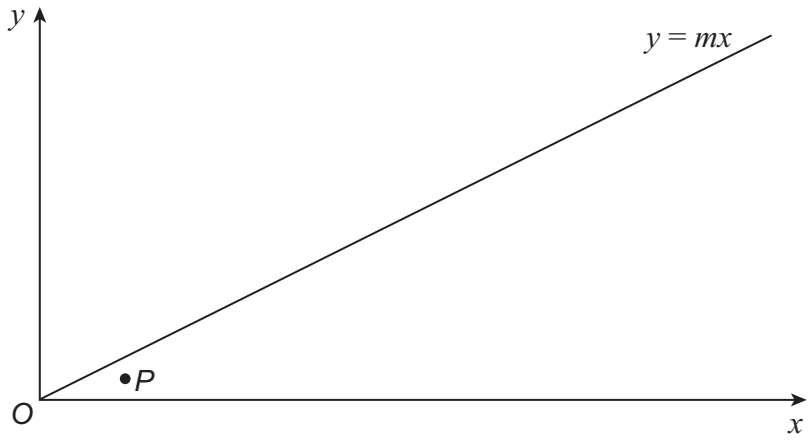
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13 (c) (i) The diagram below shows a point P and the line $y = mx$

Draw four lines on the diagram to demonstrate the result proved in part **(b)**.

Label as P' the image of P under the transformation represented by $(\mathbf{BA})^2$

[2 marks]



13 (c) (ii) Explain how your completed diagram shows the result proved in part **(b)**.

[2 marks]

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13 (d) The matrix **C** is defined as $\mathbf{C} = \begin{bmatrix} \frac{12}{5} & \frac{9}{5} \\ \frac{9}{5} & -\frac{12}{5} \end{bmatrix}$

Find the value of m such that $\mathbf{C} = \mathbf{BA}$

Fully justify your answer.

[4 marks]

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14 On an isolated island some rabbits have been accidently introduced.

In order to eliminate them, conservationists have introduced some birds of prey.

At time t years $(t \geq 0)$ there are x rabbits and y birds of prey.

At time $t = 0$ there are 1755 rabbits and 30 birds of prey.

When $t > 0$ it is assumed that:

- the rabbits will reproduce at a rate of a% per year
- each bird of prey will kill, on average, b rabbits per year
- the death rate of the birds of prey is c birds per year
- the number of birds of prey will increase at a rate of d% of the rabbit population per year.

This system is represented by the coupled differential equations:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 0.4x - 13y \qquad (1)$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 0.01x - 1.95 \qquad (2)$$

14 (a) State the value of a, the value of b, the value of c and the value of d

[2 marks]

14 (b) Solve the coupled differential equations to find both x and y in terms of t

[9 marks]

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14 (c) Given that x and y are both positive for $0 \leq t \leq 5$, use your answer to part **(b)** to show that the conservationists' plan will succeed.

[3 marks]

END OF QUESTIONS

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