AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 3 Statistics

Wednesday 14 June 2023 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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ANSWER IN THE SPACES PROVIDED

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Answer **all** questions in the spaces provided.

1 The discrete random variable A takes only the values 0, 2 and 4, and has cumulative distribution function $\mathrm{F}(a) = \mathrm{P}(A \leq a)$

a	0	2	4
$\mathrm{F}(a)$	0.2	0.6	1

Find $\mathrm{P}(A = 2)$

Circle your answer.

[1 mark]

0 0.4 0.6 0.8

2 The time, T days, between rain showers in a city in autumn can be modelled by an exponential distribution with mean 1.25

Find the distribution of the number of rain showers per day in the city.

Tick (✓) **one** box.

[1 mark]

	Distribution	Mean
☐	Exponential	0.8
☐	Exponential	1.25
☐	Poisson	0.8
☐	Poisson	1.25

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3 The masses of tins of a particular brand of spaghetti are normally distributed with mean μ grams and standard deviation 4.1 grams.

A random sample of 11 tins of spaghetti has a mean mass of 401.8 grams.

Construct a 98% confidence interval for μ, giving your values to one decimal place.

[3 marks]

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4 The random variable X has a normal distribution with unknown mean μ and unknown variance σ^2

A random sample of 8 observations of X has mean $\bar{x} = 101.5$ and gives the unbiased estimate of the variance as $s^2 = 4.8$

The random sample is used to conduct a hypothesis test at the 10% level of significance with the hypotheses

$H_0 : \mu = 100$

$H_1 : \mu \neq 100$

Carry out the hypothesis test.

[5 marks]

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5 A school management team oversees 11 different schools.

The school management team allows each student in the schools to choose one enrichment activity from 11 possible activities.

The school management team count the number of students in each school choosing each of the possible activities. They then conduct a χ^2-test for association with the data they have gathered.

5 (a) Exactly one of the calculated expected frequencies for the χ^2-test is less than 5

Explain why the number of degrees of freedom for the test is 90

[2 marks]

5 (b) The school management team claims that there is an association between the school a student attends and the activity they choose.

The test statistic is 124.8

Test the claim using the 1% level of significance.

[5 marks]

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5 (c) During the hypothesis test, the value of $\frac{(O-E)^2}{E}$, where O is the observed frequency and E is the expected frequency, was calculated for each group of students.

The values for four groups of students are shown in the table below.

Group	$\frac{(O-E)^2}{E}$
Attends school 3 and chose activity 1	0.01
Attends school 8 and chose activity 3	18.5
Attends school 8 and chose activity 7	24.2
Attends school 11 and chose activity 7	49.0

State, with a reason, which of the four groups of students represents the strongest source of association.

[1 mark]

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6 A game consists of two rounds.

The first round of the game uses a random number generator to output the score X, a real number between 0 and 10

6 (a) Find $P(X > 4)$

[1 mark]

6 (b) The second round of the game uses an unbiased dice, with faces numbered 1 to 6, to give the score Y

The variables X and Y are independent.

6 (b) (i) Find the mean total score of the game.

[3 marks]

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6 (b) (ii) Find the variance of the total score of the game.

[3 marks]

Turn over for the next question

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7 Company *A* uses a machine to produce toys.

The number of toys in a week that do not pass Company *A*'s quality checks is modelled by a Poisson distribution X with standard deviation 5

The machine producing the toys breaks down.

After it is repaired, 16 toys in the next week do not pass the quality checks.

7 (a) Investigate whether the average number of toys that do not pass the quality checks in a week has changed, using the 5% level of significance.

[7 marks]

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7 (b) For the test carried out in part **(a)**, state in context the meaning of a Type II error.

[1 mark]

7 (c) Company *B* uses a different machine to produce toys.

The number of toys in a week that do not pass Company *B*'s quality checks is modelled by a Poisson distribution Y with mean 18

The variables X and Y are independent.

Find the distribution of the total number of toys in a week produced by companies *A* and *B* that do not pass their quality checks.

[1 mark]

7 (d) State **two** reasons why a Poisson distribution may not be a valid model for the number of toys that do not pass the quality checks in a week.

[2 marks]

Reason 1

Reason 2

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8 The continuous random variable X has probability density function

$$f(x) = \begin{cases} k \sin 2x & 0 \leq x \leq \frac{\pi}{6} \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

8 (a) Show that $k = 4$

[4 marks]

8 (b) Find the cumulative distribution function $F(x)$

[3 marks]

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8 (c) Find the median of X, giving your answer to three significant figures.

[2 marks]

8 (d) Find the mean of X giving your answer in the form $\frac{1}{a}(b\sqrt{3} - \pi)$ where a and b are integers.

[5 marks]

END OF QUESTIONS

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