Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
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Centre Number | Candidate Number

# Pearson Edexcel Level 3 GCE

**Wednesday 13 October 2021 – Afternoon**

**Time** 2 hours

**Paper reference** **9MA0/02**

# Mathematics

## Advanced
## PAPER 2: Pure Mathematics 2

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question*.

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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Pearson

**1.** In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

**(2)**

(b) Hence find the sum of the first 500 terms of the series.

**(2)**

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**Question 1 continued**

**(Total for Question 1 is 4 marks)**

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**2.** The functions f and g are defined by

$$f(x) = 7 - 2x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{3x}{5x-1} \qquad x \in \mathbb{R} \qquad x \neq \frac{1}{5}$$

(a) State the range of f

**(1)**

(b) Find gf(1.8)

**(2)**

(c) Find $g^{-1}(x)$

**(2)**

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**Question 2 continued**

**(Total for Question 2 is 5 marks)**

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**3.** Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

**(3)**

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**Question 3 continued**

**(Total for Question 3 is 3 marks)**

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**4.** Given that $\theta$ is small and measured in radians, use the small angle approximations to show that

$$4 \sin \frac{\theta}{2} + 3 \cos^2 \theta \approx a + b\theta + c\theta^2$$

where $a$, $b$ and $c$ are integers to be found.

**(3)**

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**Question 4 continued**

**(Total for Question 4 is 3 marks)**

**5.** The curve $C$ has equation

$$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \qquad x \in \mathbb{R}$$

(a) Find

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(ii) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$

**(3)**

(b) (i) Verify that $C$ has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

**(4)**

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**Question 5 continued**

**(Total for Question 5 is 7 marks)**

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**6.**

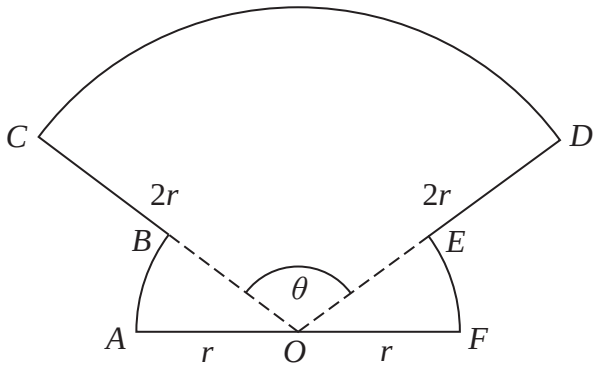



**Figure 1**

The shape *OABCDEFO* shown in Figure 1 is a design for a logo.

In the design

- *OAB* is a sector of a circle centre *O* and radius *r*
- sector *OFE* is congruent to sector *OAB*
- *ODC* is a sector of a circle centre *O* and radius 2*r*
- *AOF* is a straight line

Given that the size of angle *COD* is $\theta$ radians,

(a) write down, in terms of $\theta$, the size of angle *AOB*

**(1)**

(b) Show that the area of the logo is

$$\frac{1}{2} r^2 (3\theta + \pi)$$

**(2)**

(c) Find the perimeter of the logo, giving your answer in simplest form in terms of $r$, $\theta$ and $\pi$.

**(2)**

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**Question 6 continued**

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**Question 6 continued**

**(Total for Question 6 is 5 marks)**

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**7.** **In this question you should show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

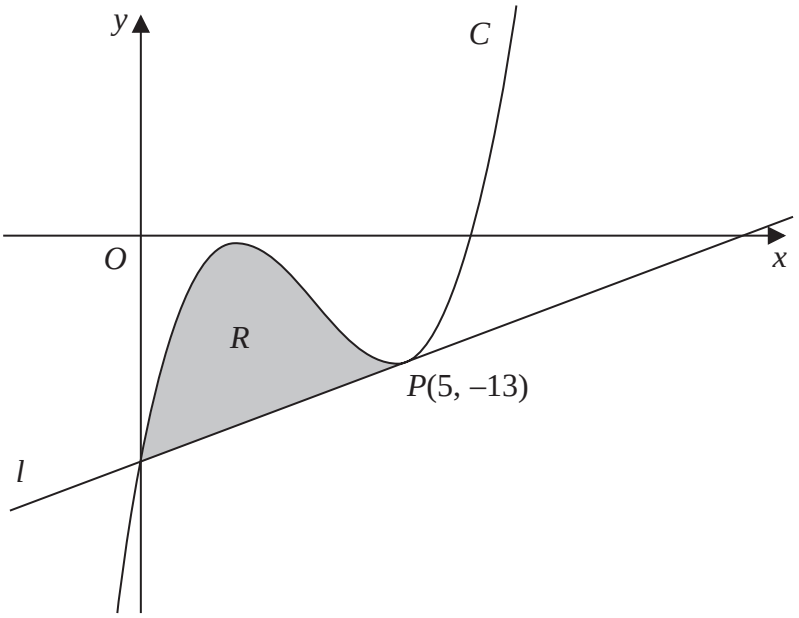



**Figure 2**

Figure 2 shows a sketch of part of the curve $C$ with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on $C$

The line $l$ is the tangent to $C$ at $P$

(a) Use differentiation to find the equation of $l$, giving your answer in the form $y = mx + c$ where $m$ and $c$ are integers to be found.

**(4)**

(b) Hence verify that $l$ meets $C$ again on the $y$-axis.

**(1)**

The finite region $R$, shown shaded in Figure 2, is bounded by the curve $C$ and the line $l$.

(c) Use algebraic integration to find the exact area of $R$.

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**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 9 marks)**

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**8.** The curve $C$ has equation

$$px^3 + qxy + 3y^2 = 26$$

where $p$ and $q$ are constants.

(a) Show that

$$\frac{dy}{dx} = \frac{apx^2 + bqy}{qx + cy}$$

where $a$, $b$ and $c$ are integers to be found.

**(4)**

Given that

- the point $P(-1, -4)$ lies on $C$
- the normal to $C$ at $P$ has equation $19x + 26y + 123 = 0$

(b) find the value of $p$ and the value of $q$.

**(5)**

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**Question 8 continued**

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**Question 8 continued**

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**Question 8 continued**

**(Total for Question 8 is 9 marks)**

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**9.** Show that

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{9}{28}$$

**(3)**

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**Question 9 continued**

**(Total for Question 9 is 3 marks)**

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**10.** The time, $T$ seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where $l$ metres is the length of the pendulum and $a$ and $b$ are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b\log_{10} l + \log_{10} a$$

**(2)**

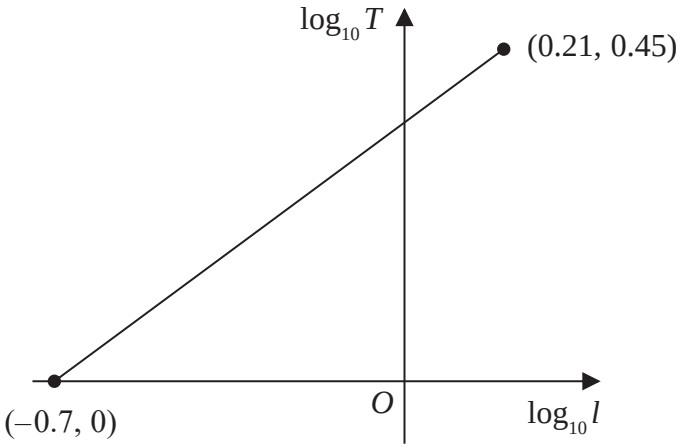



**Figure 3**

A student carried out an experiment to find the values of the constants $a$ and $b$.

The student recorded the value of $T$ for different values of $l$.

Figure 3 shows the linear relationship between $\log_{10} l$ and $\log_{10} T$ for the student's data. The straight line passes through the points (–0.7, 0) and (0.21, 0.45)

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

giving the value of $a$ and the value of $b$, each to 3 significant figures.

**(3)**

(c) With reference to the model, interpret the value of the constant $a$.

**(1)**

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**Question 10 continued**

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**Question 10 continued**

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**(Total for Question 10 is 6 marks)**

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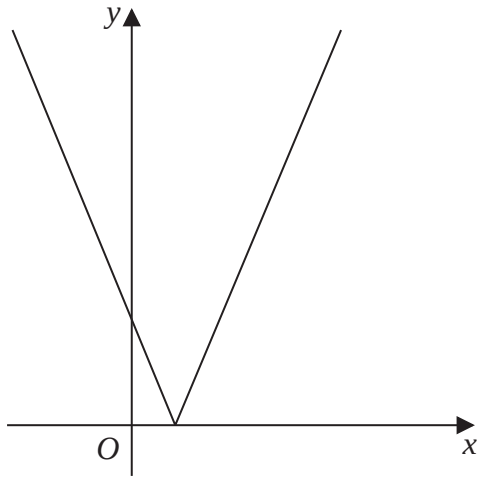



**Figure 4**

Figure 4 shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where $k$ is a positive constant.

(a) Sketch the graph with equation $y = f(x)$ where

$$f(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

**(4)**

(b) Find, in terms of $k$, the set of values of $x$ for which

$$k - |2x - 3k| > x - k$$

giving your answer in set notation.

**(4)**

(c) Find, in terms of $k$, the coordinates of the minimum point of the graph with equation

$$y = 3 - 5f\left(\frac{1}{2}x\right)$$

**(2)**

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**Question 11 continued**

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**Question 11 continued**

**(Total for Question 11 is 10 marks)**

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**12.** (a) Use the substitution $u = 1 + \sqrt{x}$ to show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,\mathrm{d}x = \int_p^q \frac{2(u-1)^3}{u}\,\mathrm{d}u$$

where $p$ and $q$ are constants to be found.

**(3)**

(b) Hence show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,\mathrm{d}x = A - B\ln 5$$

where $A$ and $B$ are constants to be found.

**(4)**

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**Question 12 continued**

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**Question 12 continued**

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**Question 12 continued**

**(Total for Question 12 is 7 marks)**

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**13.** The curve $C$ has parametric equations

$$x = \sin 2\theta \qquad y = \text{cosec}^3 \theta \qquad 0 < \theta < \frac{\pi}{2}$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of $\theta$

**(3)**

(b) Hence find the exact value of the gradient of the tangent to $C$ at the point where $y = 8$

**(3)**

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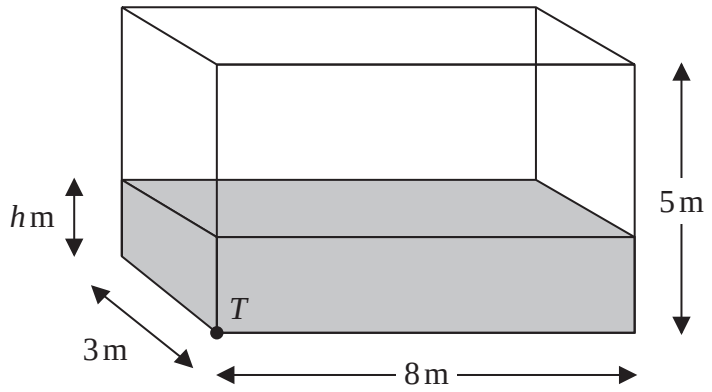



**Figure 5**

Water flows at a constant rate into a large tank.

The tank is a cuboid, with all sides of negligible thickness.

The base of the tank measures 8 m by 3 m and the height of the tank is 5 m.

There is a tap at a point $T$ at the bottom of the tank, as shown in Figure 5.

At time $t$ minutes after the tap has been opened

- the depth of water in the tank is $h$ metres
- water is flowing into the tank at a constant rate of $0.48\,\text{m}^3$ per minute
- water is modelled as leaving the tank through the tap at a rate of $0.1h\,\text{m}^3$ per minute

(a) Show that, according to the model,

$$1200\,\frac{\mathrm{d}h}{\mathrm{d}t} = 24 - 5h$$

**(4)**

Given that when the tap was opened, the depth of water in the tank was 2 m,

(b) show that, according to the model,

$$h = A + B\mathrm{e}^{-kt}$$

where $A$, $B$ and $k$ are constants to be found.

**(6)**

Given that the tap remains open,

(c) determine, according to the model, whether the tank will ever become full, giving a reason for your answer.

**(2)**

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**Question 14 continued**

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**Question 14 continued**

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**Question 14 continued**

**(Total for Question 14 is 12 marks)**

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**15.** (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of $R$ and the value of $\alpha$ in radians to 3 decimal places.

**(3)**

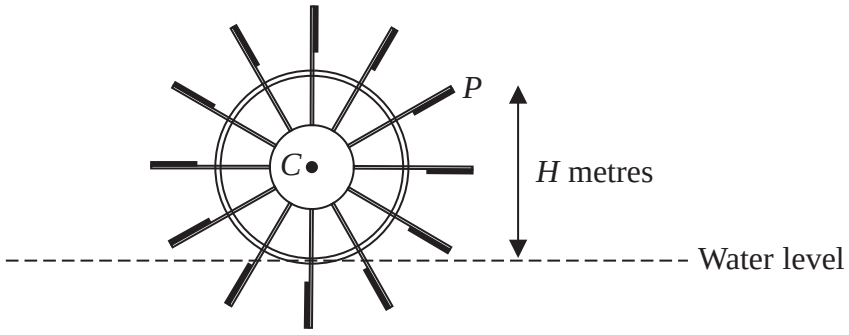



**Figure 6**

Figure 6 shows the cross-section of a water wheel.

The wheel is free to rotate about a fixed axis through the point $C$.

The point $P$ is at the end of one of the paddles of the wheel, as shown in Figure 6.

The water level is assumed to be horizontal and of constant height.

The vertical height, $H$ metres, of $P$ above the water level is modelled by the equation

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t)$$

where $t$ is the time in seconds after the wheel starts rotating.

Using the model, find

(b) (i) the maximum height of $P$ above the water level,

(ii) the value of $t$ when this maximum height first occurs, giving your answer to one decimal place.

**(3)**

In a single revolution of the wheel, $P$ is below the water level for a total of $T$ seconds.

According to the model,

(c) find the value of $T$ giving your answer to 3 significant figures.

(*Solutions based entirely on calculator technology are not acceptable*.)

**(4)**

In reality, the water level may not be of constant height.

(d) Explain how the equation of the model should be refined to take this into account.

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**Question 15 continued**

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**Question 15 continued**

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**TOTAL FOR PAPER IS 100 MARKS**

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